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Question Paper Code : 73769

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/10177 PR 401/080380009 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008/2010)

Time : Three hours   Maximum : 100 marks

(Use of statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Random Variable, and its classification.

2. Find the mgf of geometric distribution.

3. The joint pmf of two random variables X and Y is given by

$$P_{X,Y}(x,y)=\begin{cases} kxy, & x=1,2,3;\ y=1,2,3 \\ 0, & otherwise. \end{cases}$$

Determine the value of the constant k.

4. The joint pdf of a random variable (X,Y) is $f_{xy}(x,y)=xy^2+\frac{x^2}{8}$, $0\le x\le 2$, $0\le y\le 1$. Find $P\{X<Y\}$.

5. Define a Markov process.

6. Prove that the sum of two independent Poisson processes is a Poisson process.

7. Find the variance of the stationary process $\{X(t)\}$ whose auto correlation function is given by $R_{XX}(\tau)=2+4e^{-2|\tau|}$.

8. Prove that for a WSS process $\{X(t)\}$, $R_{XX}(t, t+\tau)$ is an even function of τ.

9. Check whether the system $Y(t) = X^3(t)$ is linear.

10. Compare band-limited white noise with ideal low-pass filtered white noise.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The members of a girl scout troop are selling cookies from house to house in town. The probability that they sell a set of cookies at any house they visit is 0.4.

(1) If they visit 8 houses in one evening, what is the probability that they sold cookies to exactly five of these houses?

(2) If they visited 8 houses in one evening, what is the expected number of sets of cookies they sold?

(3) What is the probability that they sold their set of cookies atmost in the sixth house they visited? (8)

(ii) Suppose X has an exponential distribution with mean equal to 10. Find the value of x such that $P(X < x) = 0.95$. (8)

Or

(b) (i) If the moments of a random variable X are defined by $E(X^r) = 0.6$, $r = 1, 2 \ldots$ then show that $P(X=0) = 0.4$, $P(X=1) = 0.6$ and $P(X \geq 2) = 0$. (8)

(ii) Find the probability density function of the random variable $y = X^2$ where X is the standard normal variate. (8)

12. (a) (i) The joint pdf of a two dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1$.

Compute $P(Y < 1/2)$, $P(X > 1 | Y < 1/2)$ and $P(X + Y \leq 1)$. (8)

(ii) If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation coefficient between $(X+Y)$ and $(X-Y)$. (8)

Or

(b) If X and Y are independent random variables with probability density functions $f_X(x) = 4e^{-4x}, x \geq 0; f_Y(y) = 2e^{-2y}, y \geq 0$ respectively, then

(i) Find the probability density functions of $U = \frac{X}{X+Y}$, and $V = X + Y$. (11)

(ii) Are U and V independent? (2)

(iii) What is $P(U > 0.5)$? (3)

13. (a) (i) Define a semi random telegraph signal process. Prove that it is evolutionary. (10)

(ii) Mention any three properties each of auto correlation and of cross correlation functions of a wide sense stationary process. (6)

Or

(b) (i) A random process $X(t)$ defined by $X(t) = A\cos t + B\sin t; -\infty < t < \infty$ where A and B are independent random variables each of which has a value -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. Show that $X(t)$ is a wide sense stationary process. (8)

(ii) If $X(t) = Y\cos\omega t + Z\sin\omega t$, where Y, Z are two independent normal random variables with $E(Y) = E(Z) = 0, Var(Y) = Var(Z) = \sigma^2$ and w is a constant, prove that $X(t)$ is a strict sense stationary process of order 2. (8)

14. (a) (i) Consider two random processes $X(t) = 3\cos(\omega t + \theta)$, and $Y(t) = 2\cos(\omega t + \theta)$, where θ is a random variable uniformly distributed over $(0, 2\pi)$. Prove that $R_{XY}(\tau) \leq \sqrt{R_{XX}(0)R_{YY}(0)}$. (8)

(ii) Find the power spectral density of a random signal with auto correlation function $e^{-\lambda|\tau|}$. (8)

Or

(b) (i) The power spectrum of a wide sense stationary process $X(t)$ is given by $S_{XX}(w) = \frac{1}{(1+w^2)^2}$. Find the auto correlation function. (8)

(ii) Find the cross correlation function corresponding to the cross-power density spectrum $S_{XY}(w) = \frac{8}{(\alpha + jw)^3}$, where $\alpha > 0$ is a constant. (8)

15. (a) If $X(t)$ is the input voltage to a circuit (system), $Y(t)$ is the output voltage, $\{X(t)\}$ is a stationary random process with $\mu_x = 0$, and $R_{xx}(\tau) = e^{-\alpha|\tau|}$ then, find μ_y, $S_{yy}(w)$ and $R_{yy}(\tau)$, if the power transfer function is $H(w) = \frac{R}{R + iLw}$, $Y(t) = \int_{-\infty}^{\infty} h(\alpha) X(t-\alpha) d\alpha$. (16)

Or

(b) (i) If the input to a time-invariant stable linear system is a wide sense process, then show that the output also is a wide sense process. (8)

(ii) If the output of the input $X(t)$ is defined by $Y(t) = \frac{1}{T} \int_{t-T}^{T} X(s)\, ds$, then show that $X(t)$ and $Y(t)$ are related by means to convolution integral. Also find the unit impulse response of the system. (8)